Further Vector Concepts

YORK COLLEGE & UNIVERSITY CENTRE

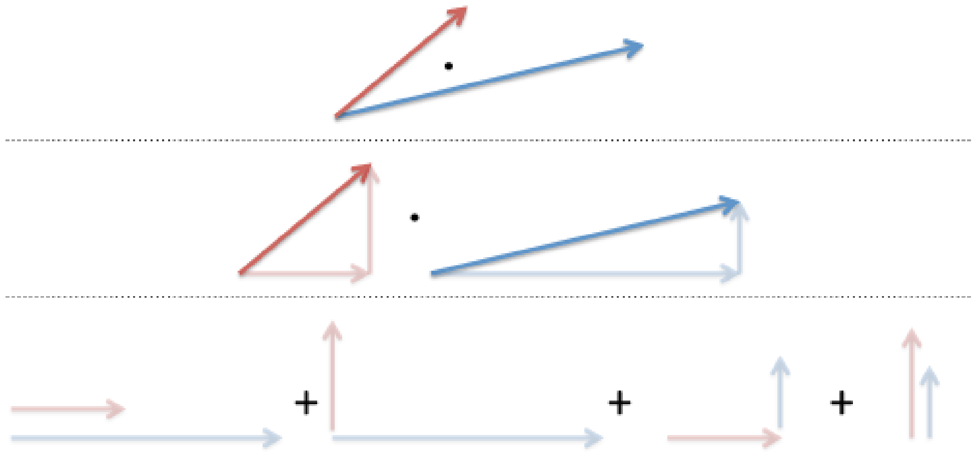
Dot Product

- Measures how much one vector acts in the direction of another

- Output is a scalar (number)

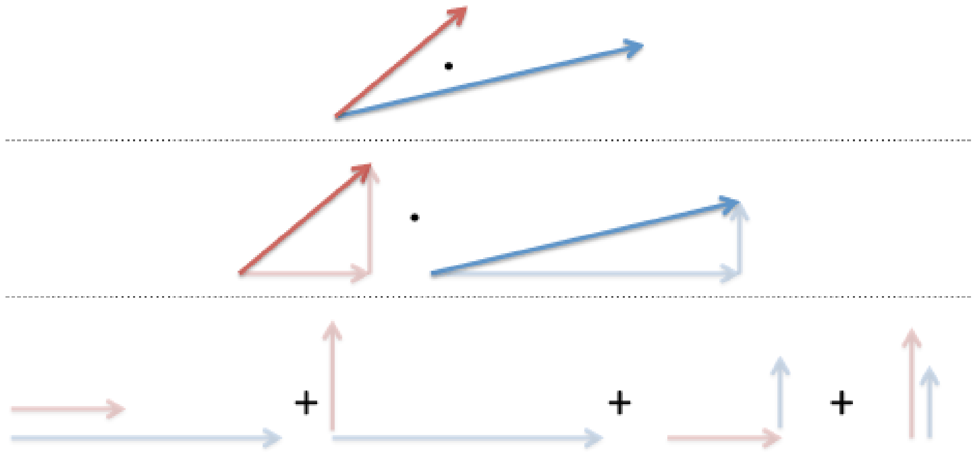
- Why it matters:

  - Work done: $W = \vec{F} * \vec{D}$
  - Power flow in electrical systems
  - Checking if vectors are perpendicular (dot = 0)



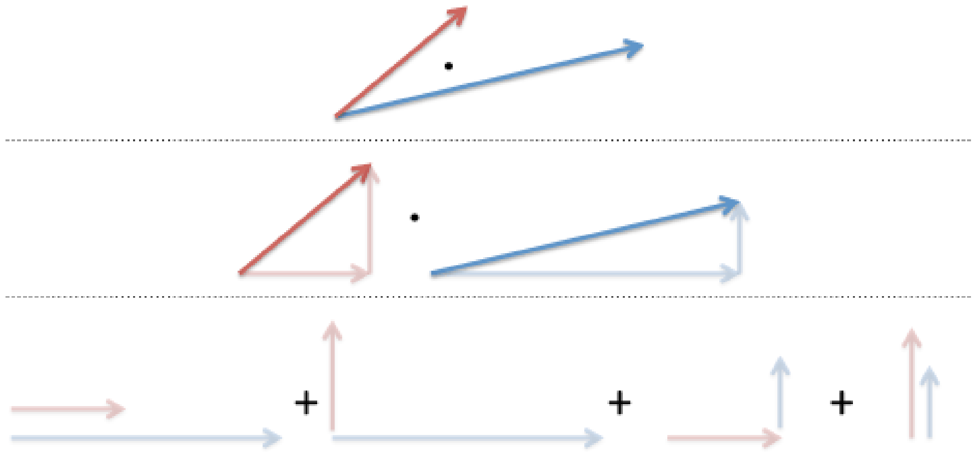
Dot Product

- There are two ways to calculate the dot product
  - $a \cdot b = |a||b| * cos\theta$
  - Or
  - $a \cdot b = (a_x * b_x) + \left(a_y * b_y\right)$



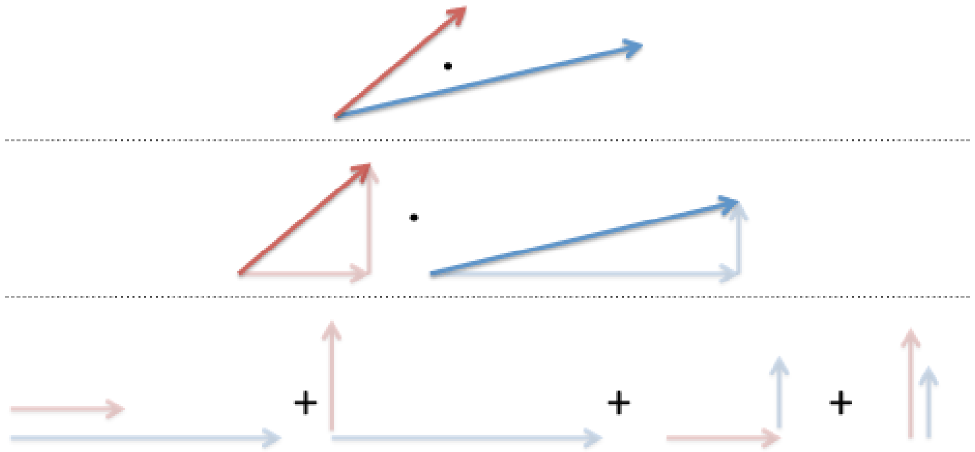
Example using Dot Product (2D)

- Say we have two forces acting on a component $\boldsymbol{F1} = (3,4)N$, $\boldsymbol{F2} = (5,0)N$
  - To work out the dot product we just do:
  - $a \cdot b = (a_x * b_x) + (a_y * b_y)$
  - $F_1 \cdot F_2 = (3 * 5) + (4 * 0) = 15$



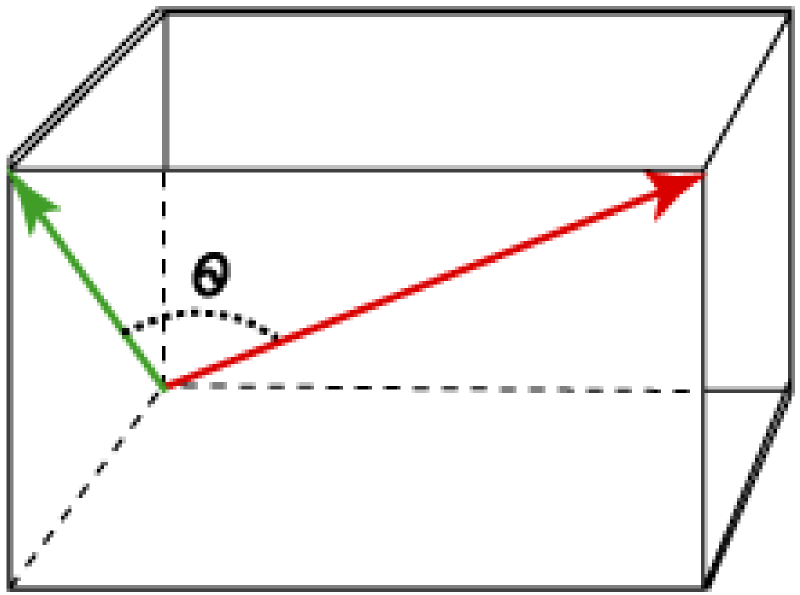
Example using Dot Product (3D)

- Two force vectors act on a joint $\boldsymbol{A} = (2, -1, 3), \boldsymbol{B} = (4, 0, -2)$
  - To work out the dot product we just do:
  - $a \cdot b = (a_x * b_x) + \left(a_y * b_y\right) + (a_z * b_z)$
  - $A \cdot B = (2 * 4) + (-1 * 0) + (3 * -2)$
  - $A \cdot B = 8 + 0 - 6 = 2$



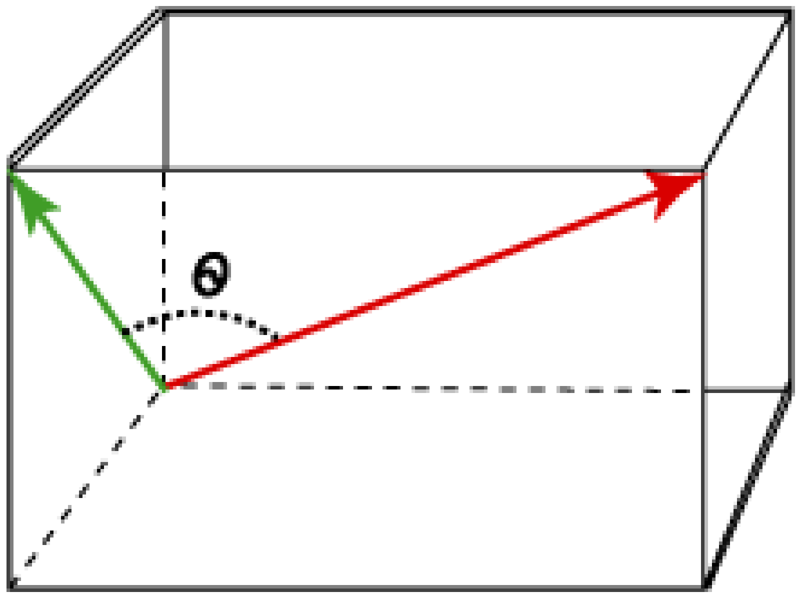
Angle Between Vectors

- If we want to work out the angle between two vectors, we need to use our dot product alongside our magnitude calculations
- The angle tells you how much one vector contributes in the direction of another.
- This can help us consider design decisions and how forces react to each other



Angle Between Vectors

- To work out the angle between two vectors we do:
- $\theta = \cos^{-1}\left(\frac{a \cdot b}{|a||b|}\right)$
- The top requires us to work out the dot product whilst the bottom requires us to work out magnitudes



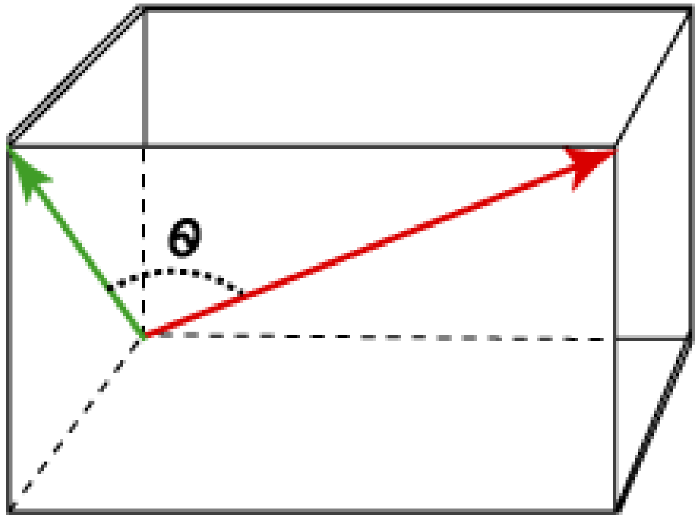
Example finding angle between (2D)

- Say we have two forces acting on a component $\boldsymbol{F1} = (3,4)N, \boldsymbol{F2} = (5,0)N$

- First, we calculate dot product: $F_1 \cdot F_2 = (3*5) + (4*0) = 15$

- Second, we calculate the magnitudes of both vectors:
  - $|F_1| = \sqrt{3^2 + 4^2} = 5$
  - $|F_2| = \sqrt{5^2 + 0^2} = 5$

- Finally, we sub into the equation:
  - $\theta = \cos^{-1}\left(\frac{a \cdot b}{|a||b|}\right) = \cos^{-1}\left(\frac{15}{5*5}\right) = \cos^{-1}\left(\frac{15}{25}\right) \approx 53.1^o$



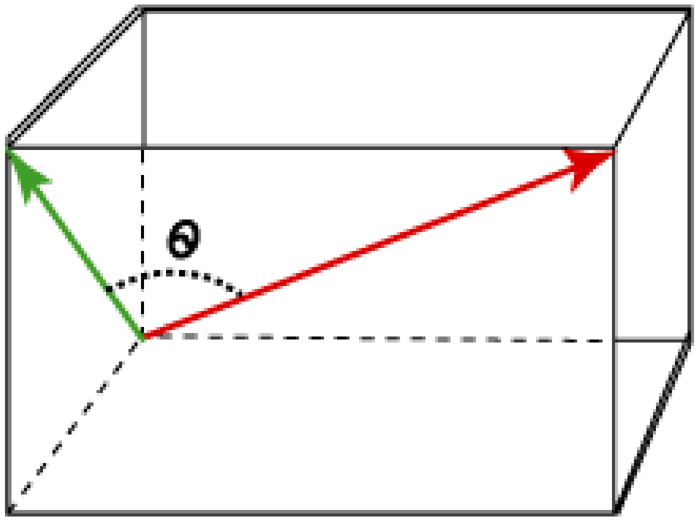
Example finding angle between (3D)

- Two force vectors act on a joint $\boldsymbol{A} = (2, -1, 3), \boldsymbol{B} = (4, 0, -2)$

- First, we calculate dot product: $A \cdot B = (2 * 4) + (-1 * 0) + (3 * -2) = 2$

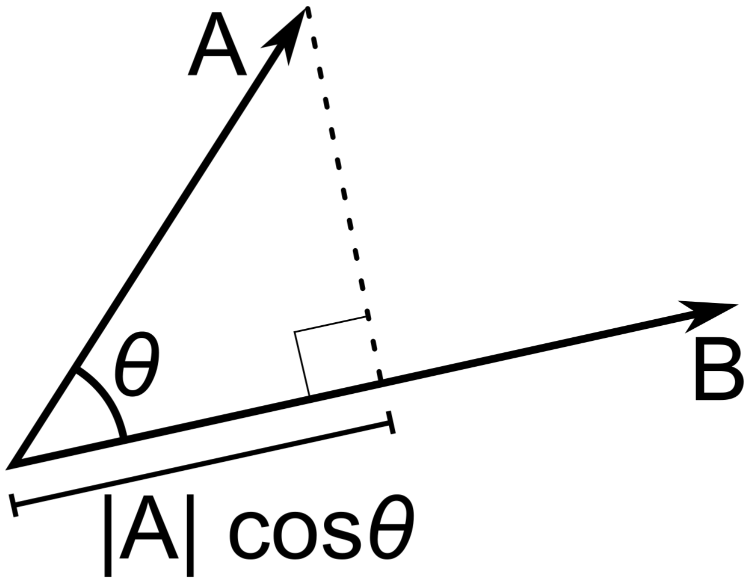
- Second, we calculate the magnitudes of both vectors:
  - $|F_1| = \sqrt{2^2 + (-1)^2 + 3^2} = \sqrt{14}$
  - $|F_2| = \sqrt{4^2 + 0^2 + (-2)^2} = \sqrt{20}$

- Finally, we sub into the equation:
  - $\theta = \cos^{-1}\left(\frac{a \cdot b}{|a||b|}\right) = \cos^{-1}\left(\frac{2}{\sqrt{14} * \sqrt{20}}\right) = \cos^{-1}\left(\frac{2}{\sqrt{280}}\right) \approx 83.1^o$



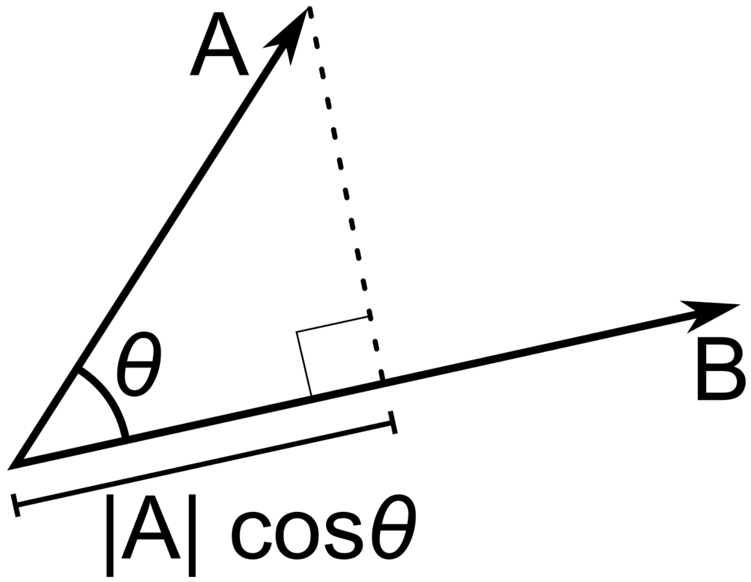
Scalar Projection

- A scalar projection tells us, "how much of F_1 acts in the direction of F_2".
- This is helpful as it allows you to determine useful components of forces, voltages and structural designs
- It essentially tells us:
- **"What part actually matters in this direction?"**



Scalar Projection

- There are two main ways we can do scalar projection:
- **A onto B** where we do:
- $\frac{\boldsymbol{A} \cdot \boldsymbol{B}}{|\boldsymbol{B}|}$
- **B onto A** where we do:
- $\frac{\boldsymbol{A} \cdot \boldsymbol{B}}{|\boldsymbol{A}|}$



Example Scalar Projection (2D)

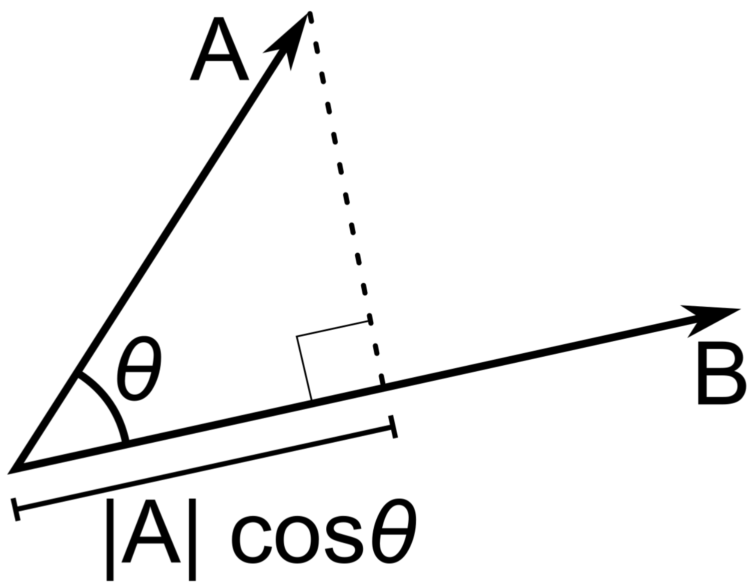
- A force is applied: $\boldsymbol{F} = (\mathbf{6}, \mathbf{8})\boldsymbol{N},$ you want the component of this force in the direction of: $\boldsymbol{d} = (\mathbf{5}, \mathbf{0})$

- $\boldsymbol{F} \cdot \boldsymbol{d} = (6 * 5) + (8 * 0) = 30$

- $|\boldsymbol{d}| = \sqrt{5^2 + 0^2} = 5$

- Then we sub into the equation:

- $comp_d(F) = \frac{F \cdot d}{|d|} = \frac{30}{5} = 6$



Example Scalar Projection (3D)

- A force is applied: $\boldsymbol{F} = (\boldsymbol{3}, -\boldsymbol{2}, \boldsymbol{6})\boldsymbol{N},$ you want the component of this force in the direction of: $\boldsymbol{d} = (\boldsymbol{1}, \boldsymbol{2}, \boldsymbol{2})$

- $\boldsymbol{F} \cdot \boldsymbol{d} = (3 * 1) + (-2 * 2) + (6 * 2) = 3 - 4 + 12 = 11$

- $|\boldsymbol{d}| = \sqrt{1^2 + 2^2 + 2^2} = \sqrt{9} = 3$

- Then we sub into the equation:

- $comp_d(F) = \frac{F \cdot d}{|d|} = \frac{11}{3} \approx 3.67$